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We extend our study of the gluon propagator in quenched lattice QCD using the Laplacian gauge to a finer lattice. We verify the existence of a pole mass as we take the continuum limit and deduce a value of $\sim 600^{+150}_{-30}$ MeV for this pole mass. We find a finite value of $(454(5) \text{ MeV})^{-2}$ for the renormalized zero-momentum propagator, in agreement with results on coarser lattices.

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I. INTRODUCTION

The gluon propagator, although not an observable quantity, plays an important role in phenomenological non perturbative studies. A framework for such studies is provided, for instance, by the Dyson Schwinger equations (DSE) which recently have been applied, among other topics, to the study of the quark gluon-plasma [1]. This application is particularly important because it complements experimental activity at RHIC and also because of the difficulties of applying lattice QCD to the case of non-zero chemical potential, although some progress is being achieved in this direction [2]. However in the Dyson-Schwinger approach the study of the gluon propagator is still inconclusive because of the various truncations needed to solve the coupled set of equations [1]. For instance, in the ghost-free axial gauge, many studies which used a simplified version of the three-gluon vertex supported an infrared enhanced gluon propagator of the form $1/q^4$. Such a behaviour, driven by the vacuum polarisation diagram, was disputed by other DSE studies [3] as being due to a flaw in setting to zero the second scalar function that enters in the definition of the gluon propagator. Similar disputes also occur in the case of the Landau gauge where some studies, which assume dominance of the gluon-vacuum polarisation, find infrared enhancement [4] whereas others [5], which use a rational polynomial Ansatz for the self energies and vertices, find an infrared vanishing propagator. After including the ghost propagator, recent studies in the Landau gauge favour an infrared-vanishing gluon propagator [6].

Lattice QCD provides a well defined theoretical framework for non perturbative physics and it is well suited for the study of the gluon propagator. A series of papers [7] have appeared over the past couple of years providing a

detailed study of the behaviour of the gluon propagator in quenched lattice QCD in the Landau gauge. However fixing to Landau gauge on the lattice via an iterative procedure may lead to ambiguous results due to the problem of lattice Gribov copies. In order to eliminate the ambiguities due to the gauge fixing we evaluated, in a previous work [8], the propagator in the Laplacian gauge which is free of lattice Gribov copies. This approach to gauge fixing on the lattice removes any doubts cast on the lattice results which were raised due to the fact that the effects of Gribov copies were unknown. We are thus in a good position to obtain physical results on quantities like the pole mass which was shown to be gauge invariant to all orders in perturbation theory [9] and compare to the corresponding values used in phenomenology. In ref. [8] it was shown that a good description of the gluon propagator was provided by an Ansatz which admits a dynamically generated gluon mass [10] and thus points to an infrared regularised gluon propagator. By analytic continuation to negative values of q^2 we obtained an estimate of the pole mass. The existence of a gluon mass has important phenomenological implications [11]. Total cross sections in hadron-hadron collisions, proton-proton elastic scattering and diffractive phenomena can be well understood if there is a finite correlation length for the gluon field [12]. For instance in the Pomeron exchange model of Landshoff and Nachtmann [13] a gluon propagator which is infrared finite is shown to eliminate the troublesome singularity in the Pomeron calculation of hadron-hadron scattering. Whereas a bare gluon mass would lead to problems with unitarity, a dynamically generated mass vanishing in the ultraviolet reproduces the correct perturbative result for the gluon propagator.

It is the purpose of the present work to check the robustness of our earlier results on the gluon propagator as we take the continuum limit. We thus extend our previ-

ous calculation on coarser lattices [8] to a finer lattice at $\beta = 6.2$. In ref. [8] we included a comparison of results in different physical volumes demonstrating that for the quantities of interest here, such as the pole mass and the zero momentum limit of the gluon propagator, a lattice size of about 1.5 fm suffices. Therefore for this study we use a lattice of spatial size ~ 1.7 fm. The quantity of interest is the transverse part, $D(q^2)$, of the propagator, the pole of which was shown to be gauge independent to all orders of perturbation theory [9]. The excellent scaling behaviour which we observe for $D(q^2)$ enables us to extract accurately the change with β of the lattice spacing. More importantly the scaling of our data in the infrared allows a global fit to the data sets at $\beta = 6.0$ and 6.2 for the extraction of the pole mass.

Our notation is the same as that of ref. [8] and we refer the reader to [8] for the details of our approach.

II. SCALING

Reasonable scaling was already observed in our previous work [8] where we compared data at $\beta = 5.8$ and 6.0 on a lattice of size $16^3 \times 32$. Here we compare data at $\beta = 6.0$ and $\beta = 6.2$.

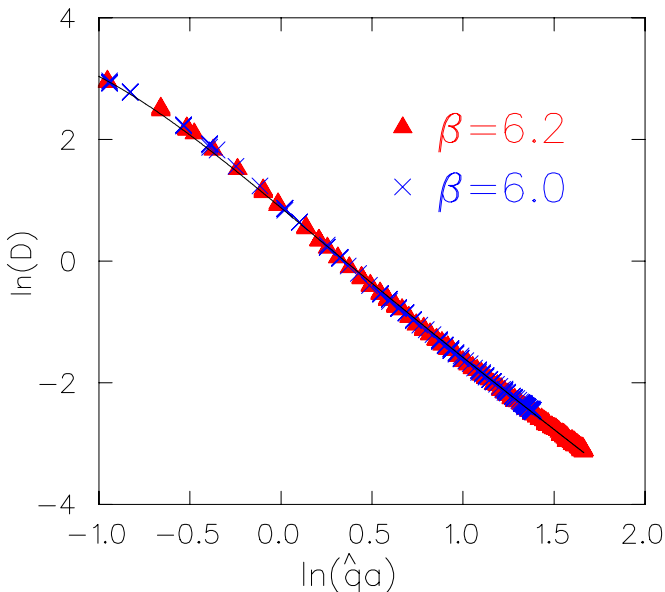


FIG. 1. Data at $\beta = 6.0$ on a $16^3 \times 32$ lattice (crosses) and at $\beta = 6.2$ on a $24^3 \times 48$ (filled triangles) fall on a universal curve (dashed line).

The results at $\beta = 6.2$ were obtained by using 220 configurations generated by the UKQCD collaboration on a lattice of size $24^3 \times 48$. At $\beta = 6.0$ we used 200 configurations of size $16^3 \times 32$ from the NERSC archive [14]. Being now closer to the continuum limit, we find very

good scaling behaviour as demonstrated in Fig. 1 where the two sets of data fall on the same curve.

Scaling ratios are extracted by comparing these two sets of results. In Fig. 1 the two sets of data are shifted according to

$$\ln(D_{\beta=6.0}(\ln qa_{\beta=6.0})) = \ln(D_{\beta=6.2}(\ln(qa_{\beta=6.2} - b)) + c \quad (1)$$

We find $b = 0.277 \pm 0.022$ and $c = -0.574 \pm 0.053$ yielding

$$\begin{aligned} \frac{Z_{\beta=6.2}}{Z_{\beta=6.0}} &= 1.02 \pm 0.14 \\ \frac{a_{\beta=6.2}}{a_{\beta=6.0}} &= 0.758 \pm 0.017 \quad . \end{aligned} \quad (2)$$

This ratio of lattice spacings is consistent with that obtained from measurements of the string tension [15], and very close to the value 0.729 obtained from the interpolation formula of the Alpha-collaboration [16] for r_0/a .

The renormalised zero momentum propagator also exhibits good scaling and we obtain a value of $(454(5)\text{MeV})^{-2}$ in agreement with our previous value of $\sim (445\text{MeV})^{-2}$, where we took our renormalization point at 1.943 GeV, and our lattice spacing $a^{-1}(\beta = 6.2) = 2.718$ GeV [15].

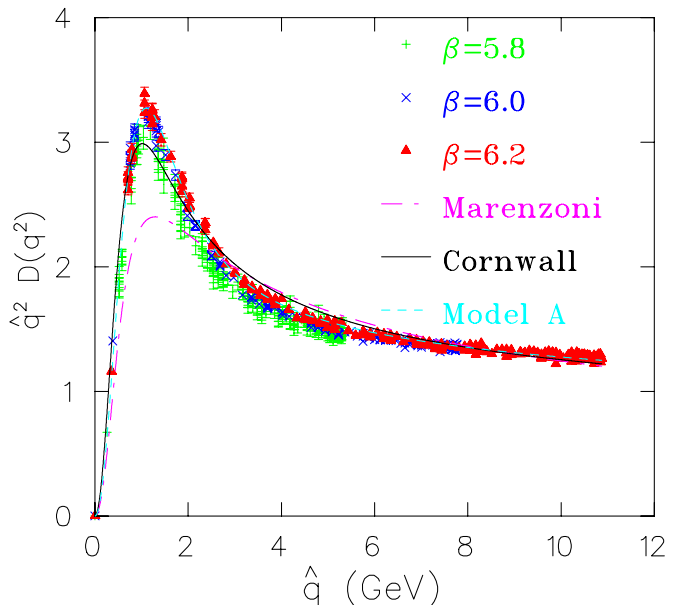


FIG. 2. The transverse gluon propagator multiplied by q^2 at $\beta = 5.8, 6.0$ and 6.2 in physical units. The fits to three models are shown: Marenzoni (dashed dotted line) [17], Cornwall (solid line) [10], and model A (dashed line) of ref. [7]

Our new data for the transverse propagator at $\beta = 6.2$ are shown in Fig. 2 together with our previous results at $\beta = 6.0$ and 5.8. From the various proposals put forward for the gluon propagator the three that yield the best fits are due to Marenzoni *et al.* [17], Cornwall [10],

and Model A of ref. [7] which has four parameters instead of three like the other two. Cornwall's ansatz is not a simple parametrization like the others, but is derived consistently from a physical model. As we did on coarser lattices, we find again at $\beta = 6.2$ that Cornwall's Ansatz provides a good fit to the data for the whole momentum range. We will thus use it to extrapolate to negative values of q^2 for the determination of the pole mass.

III. POLE MASS

As explained in the Introduction, a phenomenologically important question is whether the gluon propagator has a pole mass. We show in Fig. 3 our data at $\beta = 6.2$ and $\beta = 6.0$ in physical units which nicely fall on a universal curve, especially after performing a “cylindrical cut” in momenta which removes data most affected by lattice artifacts [7].

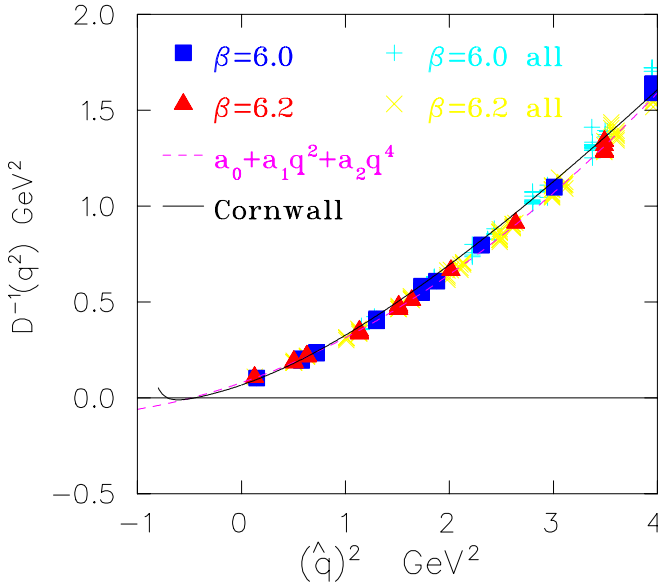


FIG. 3. The inverse gluon propagator at low momentum, at $\beta = 6.0$ (pluses) and 6.2 (crosses). The filled symbols show the data which are kept after the cylindrical momentum cut [7]. Two extrapolations to negative \hat{q}^2 are shown: quadratic polynomial in \hat{q}^2 (dashed line), and Cornwall's model (solid line).

This allows us to perform a simultaneous fit to both sets of data, using a quadratic polynomial in q^2 as well as Cornwall's Ansatz. Analytically continuing Cornwall's Ansatz to negative values of q^2 we find a pole mass of 669(6) MeV. The pole mass extracted from the quadratic polynomial is 693(20) MeV. Fig. 4 compares the results at the three different lattice spacings for these two Ansätze. It can be seen that both Cornwall's Ansatz and the quadratic polynomial yield consistent results. If

a continuum extrapolation in a^2 is performed from the data at $\beta = 5.8, 6.0$ and 6.2 , we obtain pole masses of 632(38) MeV and 592(14) MeV using the quadratic polynomial and Cornwall's Ansatz respectively.

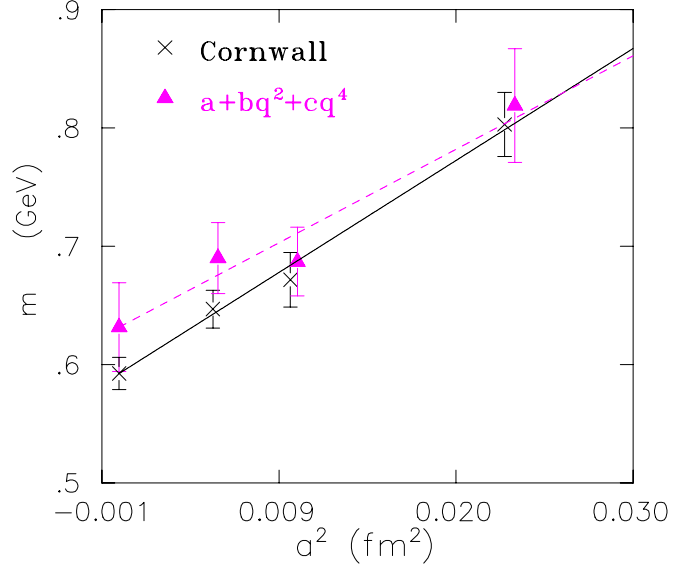


FIG. 4. The pole mass at $\beta = 5.8, 6.0$ and 6.2 extracted from fitting to a quadratic polynomial (shifted to the right for clarity) and to Cornwall's Ansatz, as a function of the lattice spacing a^2 . The continuum values are obtained by linear extrapolation in a^2 .

From these data we can conclude for strong evidence that a pole exists which survives the continuum limit, with a mass of about 600 MeV. Because of the curvature of the inverse propagator, the systematic error in the extrapolation to negative q^2 is asymmetric. We estimate it conservatively at $\sim -30, +150$ MeV. Based on our study of finite volume effects [8], and given our present lattice size, we expect negligible finite size corrections to the pole mass. Comparing it to the glueball mass of 1.73(0.05)(0.08) GeV [18], it appears that the pole mass of the gluon propagator is close to one third of the glueball mass (rather than one half as sometimes speculated). A pole mass of 500 – 800 MeV is also within the range needed to fit experimental data in various phenomenological studies [11,12].

A further check on the value of the pole mass is provided by measuring the correlator of the gluon field averaged over a time-slice. Namely, we measure [8]

$$C(t) = \frac{1}{L_s^3} \frac{1}{8} \sum_{a=1}^8 \frac{1}{3} \sum_{\mu=1}^3 \left(\sum_x A_\mu^a(\vec{x}, 0) \right) \left(\sum_x A_\mu^a(\vec{x}, t) \right) \quad (3)$$

This correlator is displayed in Fig.5 where the rate of exponential decay gives a model-independent determination of the pole mass. Although this correlator is measured on the same configurations as $D(q^2)$, the various

momenta are given a different weight, so that a fit to $C(t)$ will give different results than a fit to $D^{-1}(q^2)$, especially after the cylindrical momentum cut in the latter. Therefore, we also fit Cornwall's model directly to $C(t)$ instead of $D^{-1}(q^2)$. The dashed line in Fig.5 shows the original fit of Cornwall's ansatz to $D^{-1}(q^2)$, which already provides a fair description of the data. The solid line represents a direct fit of the same 3-parameter ansatz to $C(t)$, excluding the first few time-slices which are contaminated by contributions from excited states.

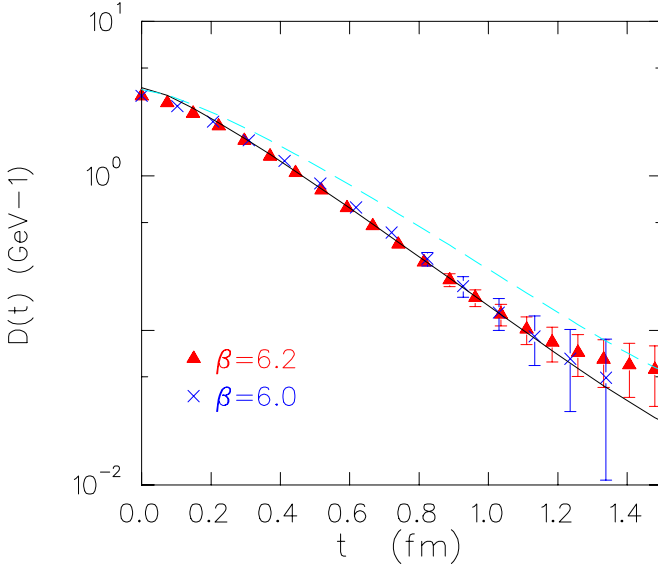


FIG. 5. Time-slice gluon correlator at $\beta = 6.0$ and 6.2 . The dashed line shows Cornwall's model fitted to $D^{-1}(q^2)$ after the cylindrical momentum cut; the solid line is a direct fit of the same model to the time-slice correlators, excluding the first few time-slices.

A simultaneous fit of the time-slice correlator data at $\beta = 6.2$ and 6.0 yields a pole mass of $739(81)$ MeV, in agreement with the value $669(6)$ MeV extracted from the momentum propagator after a cylindrical momentum cut.

The pole mass can also be extracted in a model-independent way, from the asymptotic decay rate of the time-slice correlator. From the effective mass $m_{eff}(t) = -\ln(C(t+1)/C(t))$, we obtain a value of $702(163)$ MeV, consistent with the direct fit. Therefore, all our different analyses give pole masses ranging from 592 MeV (continuum extrapolation of pole of propagator fitted by Cornwall's Ansatz) to 739 MeV (fit of time-slice correlator), where we have used $a^{-1}(\beta = 6.2) = 2.718$ GeV to convert to physical units [15] with $\sqrt{\sigma} = 440$ MeV. Taking into account the asymmetry of potential systematic errors, we estimate the gluon pole mass to be $\sim 600^{+150}_{-30}$ MeV.

IV. CONCLUSIONS

We have extended a previous lattice study of the gluon propagator in the Laplacian gauge to a finer lattice and found good scaling behaviour. We confirm the existence of a pole as we approach the continuum limit. Applying a variety of different fits we extract a pole mass in the range of $600 - 30 + 150$ MeV in accord with the value found in phenomenological studies for the description of hadron-hadron scattering.

The question of a gluon mass was also addressed recently in ref. [19] where a non-local gauge invariant gluon propagator was constructed and first numerical results were obtained in $(2+1)$ dimensions for $SU(2)$. It was argued there that the pole mass determines the vector-pseudoscalar-mass splitting, $M_V - M_S$, in heavy quarkonia. This relation was verified by the numerical results. For QCD, using the experimental values for the splitting in $c\bar{c}$ and $b\bar{b}$ systems, the implication is that the pole mass is ~ 420 MeV which is about one quarter of the glueball mass. In contrast we find a value close to one third.

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